Models for Inexact Reasoning

Fuzzy Logic – Lesson 4
Fuzzy Hedges

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Fuzzy Hedges

• Hedges are special terms aimed to modify other linguistic terms
• Can be used to modify elements such as:
  – Fuzzy predicates
  – Fuzzy truth values
  – Fuzzy probabilities
• Examples:
  – “Very”, “more or less”, “fairly”, “extremely”, etc.
Fuzzy Hedges – Examples

• Modification of a fuzzy predicate
  – “x is very young”
• Modification of a fuzzy truth value
  – “x is young is very true”
• Modification of a fuzzy probability
  – “x is young is very likely”
• Modification of both a predicate and a truth value
  – “x is very young is very true”
Linguistic Modifiers

- Not applicable to crisp predicates, truth values and probabilities
- Given a fuzzy proposition $p$, and a hedge $H$: $Hp: x \text{ is } HF$
- Hedges are represented by unary operations on $[0, 1]$
- These operations are called “modifiers”
- Example: $h(a) = a^2$ is a modifier representing the hedge “very”
Linguistic Modifiers

• Given a fuzzy predicate $p$: “$x$ is $F$” and a hedge $H$ represented by modifier $h$ we have:

\[ \mu_{HF}(x) = h(\mu_F(x)) \]

• Any modifier $h$ is an increasing bijection
  – Strong modifiers: $h(a) < a$, $\forall a \in [0,1]$
  – Weak modifiers: $h(a) > a$, $\forall a \in [0,1]$
  – Identity modifier: $h(a) = a$, $\forall a \in [0,1]$
Strong and Weak Modifiers

- Strong modifiers “strengthen” the predicate
  - Reduce the truth value of the associated proposition
- Weak modifiers “weaken” the predicate
  - Increase the truth value of the associated proposition
- The identity modifier has no effect
  - The truth value of the associated proposition remains unchanged
Example

- **Predicates:**
  - \( p_1 \): “John is young”
  - \( p_2 \): “John is very young”
  - \( p_3 \): “John is fairly young”
- **Hedges:**
  - \( H_1 \): very, \( h_1(x) = a^2 \)
  - \( H_2 \): fairly, \( h_2(x) = \sqrt{a} \)
- **Age(John) = 26**
- **T(\( p_1 \))? T(\( p_2 \))? T(\( p_3 \))?**
Properties of modifiers

- $h(0) = 0$
- $h(1) = 1$
- $h$ is continuous
- If $h$ is strong, then $h^{-1}$ is weak (and vice versa)
- If $h_1, h_2$ are modifiers, then $(h_1 \circ h_2)$ and $(h_2 \circ h_1)$ are modifiers
- If both $h_1$ and $h_2$ are strong (weak) modifiers then so are the compositions
A Family of Modifiers

- A class of functions that satisfies the previous conditions:
  \[ h_\alpha(a) = a^\alpha, \quad \alpha \in \mathbb{R}^+ \]

- If \((\alpha < 1)\) then \(h_\alpha\) is a weak modifier.
- If \((\alpha > 1)\) then \(h_\alpha\) is a strong modifier.
- \(h_1\) is the identity modifier.
- We can choose a suitable value for \(\alpha\) depending on the context.
The antonym

• All fuzzy predicates have an antonym:

Definition of antonym

Overview of *noun* antonym

The noun antonym has 1 senses? (no senses from tagged texts)

1. antonym, opposite word, opposite
(two words that express opposing concepts; "to him the opposite of gay was depressed")

• Example:
  – p: “x is tall”
  – ap: “x is short”
The antonym

• Do not confuse the antonym with the negation!!
  – The negation of “tall” is “not tall” instead of “short”

• If E is a continuous interval \([a, b]\), then the antonym is calculated as follows:

\[
\mu_{aF}(x) = \mu_F(a + b - x), \quad x \in [a, b]
\]
Example

• John is 50 years old
• $T(\text{“John is not young”})$?
• $T(\text{“John is old”})$?
• $T(\text{“John is neither very young nor fairly old”})$?