Fuzzy Quantifiers

• The scope of fuzzy propositions can be extended using fuzzy quantifiers
• Fuzzy quantifiers are fuzzy numbers that take part in fuzzy propositions
• There are two different types:
  – Type #1 (absolute): Defined on the set of real numbers
    • Examples: “about 10”, “much more than 100”, “at least about 5”, etc.
  – Type #2 (relative): Defined on the interval [0, 1]
    • “almost all”, “about half”, “most”, etc.
Quantified Propositions

• Propositions involving type #1 quantifiers:
  \[ p: \text{There are } Q \text{ i’s in } I \text{ such that } V(i) \text{ is } F \]
• \( Q \) is a type #1 quantifier
• \( i \) is an individual from a given set \( I \)
• \( V(i) \) is a variable associated to the individual \( I \) that takes values from a universe \( E \)
• \( F \) is a fuzzy set defined over the universe \( E \)
Quantified Propositions

- Example:
  “There are about 10 students in a given class whose fluency in English $V(i)$ is high”

- $Q$ = “about 10”
- $i$’s = students
- $I$ = class
- $V(i)$ = Fluency in English of student $i$
- $F$ = Fuzzy set that represents a high degree of fluency in English
Quantified Propositions

• The former propositions can be converted to a simpler form:
  \[ p' : \text{There are } Q \text{ } Z's \text{ in } I \]

• Z is a fuzzy set defined as follows:
  \[ \mu_z(i) = \mu_F(V(i)) \quad \forall i \in I \]

• Thus, the former proposition is replaced with:
  “There are \underline{about 10} high-fluency English-speaking students in a \underline{given} class”
Quantified Propositions

• To calculate the truth value of $p'$ we need to calculate the cardinality of fuzzy set $Z$

• The cardinality of $Z$ can be calculated as follows:

$$|Z| = \sum_{i \in l} \mu_z(i) = \sum_{i \in l} \mu_F(V(i))$$

• Then, we calculate the truth value $T(p')$ using the membership function of $Q$

$$T(p') = \mu_Q(|Z|)$$
Example

“There are about 3 students in \( I \) whose fluency in English \( V(i) \) is high”

- \( I = \{ \text{Adam, Bob, Cathy, David, Eve} \} \)
- \( V = \text{Degree of fluency in English} \)
  - \( V(\text{Adam}) = 35, V(\text{Bob}) = 20, V(\text{Cathy}) = 80, V(\text{David}) = 99, V(\text{Eve}) = 70 \)
Example

[Graph showing a cumulative distribution function with labels for Bob, Adam, Eve, Cathy, and David. The function is labeled as F: “high fluency”]
Example

Q(\|E\|) = 0.625

Q: “about 3”
Quantified Propositions

- Type #1 fuzzy quantifiers may also appear in more complex propositions:
  \[ p: \text{There are } Q \text{ i's in } I \text{ such that } V_1(i) \text{ is } F_1 \text{ and } V_2(i) \text{ is } F_2 \]

- Example:
  “There are at least about two students in the class whose fluency in English is high and are young”
Quantified Propositions

- Alternative format for propositions of the former type:
  \[ p' \colon \text{There are Q } Y \text{ in I, } \]
  \[ Y = \text{T-norm}(\mu_{F1}(V_1(i)), \mu_{F2}(V_2(i))) \]

- Example:
  \[ p' \colon \text{“There are at least about two high-fluency English-speaking and young students in the class”} \]

- Obviously Y is the set of “English-speaking and young students”
Exercise (Homework)

• Calculate the truth value of the proposition:
  “There are at least about 3 students in the class whose fluency in English is high and are young”
• Use the data in slide #7 (students) for: I, V(i), μ_Q and μ_F
• Use the following T-norm: T(x, y) = min(x, y)
• The ages for the different students are:
  – Age(Adam)=23, Age(Bob)=35, Age(Cathy)=46,
  – Age(David)=54, Age(Eve)=25
Exercise (Homework)

- Membership function of fuzzy set “Young”
Quantified Propositions

- It is also possible to have propositions involving type #2 quantifiers:

  \[ p: \text{Among i’s in I such that } V_1(i) \text{ is } F_1 \text{ there are Q i’s in I such that } V_2(i) \text{ is } F_2 \]

- Example:

  “Among students in a given class that are young, there are almost all whose fluency in English is high”
Quantified Propositions

• This kind of propositions can be rewritten as:
  \[ p': Q \text{ Y's are Z's} \]
  \[ Y = \mu_{F_1}(V_{F_1}(i)), \ Z = \mu_{F_2}(V_{F_2}(i)) \]

• Example:
  "Almost all young students in a given class are students whose fluency in English is high"

• How do we calculate the truth value of such propositions?
Quantified Propositions

• We can rewrite \( p' \) as \( p'' \): \( W \) is Q
• \( W \) is the degree of subsethood of \( Y \) in \( Z \)
  – How do we calculate \( W \)?

\[
W = \frac{|Z \cap Y|}{|Z|} = \frac{\sum_{i \in I} \min \left( \mu_{F_1} (V_1 (i)), \mu_{F_2} (V_2 (i)) \right)}{\sum_{i \in I} \mu_{F_1} (V_1 (i))}
\]

• Once we have obtained \( W \), we can easily calculate \( T(p) \)

\[
T ( p ) = \mu_Q ( W )
\]
Qualified Propositions

- Some examples of type #2 quantifiers:
Exercise (Homework)

• Calculate the truth value of the sentence:
  “Almost all young students in a given class are students whose fluency in English is high”

• Use the data and membership functions from the previous exercise

• Use the membership function given in the previous slide for the relative quantifier “almost all”